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POTTS MODEL ON THE BETHE LATTICE WITH NONMAGNETIC IMPURITIES IN AN EXTERNAL MAGNETIC FIELD

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We obtain a solution for the Potts model on the Bethe lattice in an external magnetic field with movable nonmagnetic impurities. Using the method of "pseudochaotic" impurity distribution (correlations in the positions of the impurity atoms for the neighboring sides vanish), we obtain a system of equations defining the first-order phase transition curve on the "temperature–external field" plane. We find the dependence of the endpoint of the phase transition line on the concentration of magnetic atoms.

Keywords: diluted magnet, Potts model, Bethe lattice, phase transition

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1. Introduction

The Potts model [1] is one of the most often used models in statistical [2], [3] and nuclear physics [4]–[6]. The phase transition in the Potts model without an external field has often been considered for both a pure magnet [1]–[4] and a magnet with nonmagnetic impurities [7], [8]. But the critical behavior of the Potts model in an external field is also interesting [5], [6]. There are materials (such as $SrTiO_2$) in which structural phase transitions are related to the universality class of the Potts model with three states [2].

In addition, the Potts model is a basis for the theoretical description of complicated anisotropic ferromagnets with a cubic structure, multialloys, and liquid mixtures [3].

Here, we consider the Potts model with a nonmagnetic dilution (over the sites) and an arbitrary number of states on the Bethe lattice in an external field. To take the effect of nonmagnetic dilution into account, we use the method of a pseudochaotic distribution of impurities presented in [8]. The idea of the method is to use the condition of zero correlation in impurity positions for neighboring lattice sites in the problem with movable nonmagnetic impurities. This problem was previously considered in [9] using the mean-field method and its modifications. In this approximation, the lines of first-order phase transitions on the temperature–external field plane were obtained depending on the concentration of the magnetic atoms, and the magnetic susceptibility near the phase transition point was calculated. But in calculations using the mean-field method, the phase transition appears to be possible for any concentration of the magnetic atoms, which contradicts the presence of a percolation threshold in real lattices. Unlike the mean-field method, when the problem is solved using the pseudochaotic approximation, the percolation transition exists for a zero concentration of magnetic atoms, as is shown below.

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2. Potts model on the Bethe lattice with a pseudochaotic impurity distribution

The model can be formulated as follows. We consider the Bethe lattice with the coordination number q. We associate each site of the lattice with a quantity σ_i that can take s different values $1, 2, \ldots, s$ if there is a magnetic atom at the site. Two neighboring magnetic atoms interact with the energy $-J_\mathrm{p}\delta(\sigma_i, \sigma_j)$, where

$$\delta(\sigma_i, \sigma_j) = \begin{cases} 1, & \sigma_i = \sigma_j, \\ 0, & \sigma_i \neq \sigma_j. \end{cases}$$

We assume that nonmagnetic atoms ("impurities") can be at some sites of the lattice. We also assume that b is the fraction of the magnetic atoms and $1-b$ is consequently the fraction of impurities in the lattice. Let σ_i take the value zero if there is a nonmagnetic impurity at the site. We assume that the interaction forces act only between neighboring atoms and there is an external field H that acts on the state 1. The contribution to the system energy from two neighboring sites can then be represented as

$$\begin{aligned} E_{ij} = {} & -H(\delta(\sigma_i, 1) + \delta(\sigma_j, 1)) - J_\mathrm{p}\delta(\sigma_i, \sigma_j) - (U_{11} - J_\mathrm{p})\delta(0, \sigma_j)\delta(\sigma_i, 0) - \\ & - U_{12}\{\delta(\sigma_i, 0)(1 - \delta(0, \sigma_j)) + \delta(0, \sigma_j)(1 - \delta(\sigma_i, 0))\} - \\ & - U_{22}(1 - \delta(0, \sigma_j))(1 - \delta(\sigma_i, 0)). \end{aligned}$$

Here, U_{11} is the interaction energy between two neighboring impurity atoms, U_{12} is the interaction energy of an impurity atom with a magnetic atom, and U_{22} is the interaction energy between two magnetic atoms.

The grand partition function of the system is

$$Z = \sum \exp\biggl\{K \sum_{(i,j)} \phi(\sigma_i, \sigma_j) + h \sum_i^{\delta} (\sigma_i, 1) + x \sum_i^{\delta} (\sigma_i, 0)\biggr\}, \tag{1}$$

where $K = J_\mathrm{p}/kT$, $h = H/kT$, $x = \mu/kT$ (μ is the chemical potential), and

$$\phi(\sigma_i, \sigma_j) = \delta(\sigma_i, \sigma_j) + (\gamma - 1)\delta(0, \sigma_j)\delta(\sigma_i, 0), \qquad \gamma = \frac{U}{J_\mathrm{p}}, \qquad U = U_{11} - 2U_{12} + U_{22}.$$

Using expression (1) for pseudochaotically distributed impurities, we can calculate the probability p_1 that a magnetic atom at a lattice site is in state 1 [8]:

$$p_1 = b^2 \frac{e^{K+2h} + te^h + (n-1)e^h y^{q-1}}{e^{K+2h} + (n-1)y^{q-1}(2e^h + y^{q-1}(e^K + n - 2))}, \tag{2}$$

where

$$t = \frac{1-b}{b} \frac{e^{K+2h} + (n-1)y^{q-1}(2e^h + y^{q-1}(e^K + n - 2))}{e^h + (n-1)y^{q-1}} \tag{3}$$

and

$$y = \frac{t + e^h + (e^K + (n-2))y^{q-1}}{t + e^{K+h} + (n-1)y^{q-1}}. \tag{4}$$

System (2)–(4) was analyzed in [8] in the particular case $h = 0$ (without an external field). In particular, it was shown there that Eqs. (2)–(4) describe the phase transition (first order if $s > 2$ and second order if $s = 2$) under the condition $b > 1/(q-1)$, i.e., when the concentration of magnetic atoms is above the